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ON THE ESTIMATION OF PERTURBATIONS DUE TO FLOW AROUND BLAST GAUGES

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Approved for Distribution
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ON THE ESTIMATION OF PERTURBATIONS DUE TO FLOW AROUND BLAST GAUGES

1. Introduction

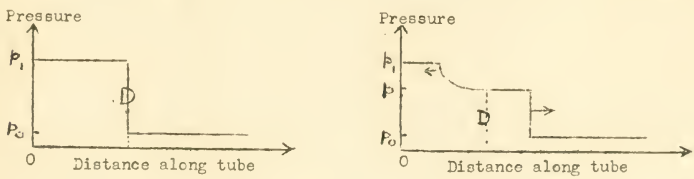
A shock tube[1] developed at the Princeton University Station of Div. 2 (NDRC) furnishes a method for the calibration of tourmaline gauges (and other gauges for measuring pressure) under dynamic conditions resembling those in air blasts from bombs and guns. Some anomalies in the calibrations have been observed[2] and have been attributed to the deflection of the air flow around the gauges. This memorandum presents the results of a theoretical investigation on the magnitude of such effects and offers criteria for choosing sizes and shapes of baffles (and of shock tubes) to reduce the pressure perturbations to negligible proportions.

2. Physical Considerations

The shock tube is a pipe, generally of rectangular cross-section, sealed at one or both ends and divided by a cellophane diaphragm D (see Fig. 1) into a compression chamber C and an expansion chamber E.

FIGURE 1. Diagram of the shock tube.

An initial pressure discontinuity is established by forcing air into C (or evacuating air from E). The diaphragm D is then broken and a nearly plane shock wave travels into E while a rarefaction wave travels into C as indicated in Fig. 2.



Pressure distribution before the diaphragm D is broken.

Pressure waves after the diaphragm D is broken.

FIGURE 2.

The pressure distribution can be calculated explicitly (see Appendix).

If a "stiff" pressure gauge is mounted with its pick-up face flush with the wall of the tube (see G in Fig. 1) then, after the shock wave has passed over, the face is exposed to a constant pressure p until reflections come from the ends of the tube. In this arrangement the wall of the tube acts as a baffle for the gauge. If, however, the gauge is mounted in the center of the tube in the "edge on" position with respect to the oncoming shock wave, the gauge will distort the shock wave and, more importantly, the flow behind the shock wave. This exposed end-on position has been favoured (at least until recently) because it corresponds to the orientation employed in air blast measurements.

In a series of experiments Arons, et al [2] have found that the shock tube calibrations of tourmaline-crystal pressure gauges in the exposed position indicate that the gauge perturbs the pressure p, the perturbation becoming larger for larger values of p_1/p_0 (see Fig. 2).

It has been suggested that this perturbation is mainly due to the distortion produced in the mass flow pattern by the presence of the gauge itself. If this hypothesis is correct, the resulting pressure change could be reduced by placing a properly designed baffle around the gauge as indicated in Fig. 3, and by having the cross-section of the shock tube sufficiently large.

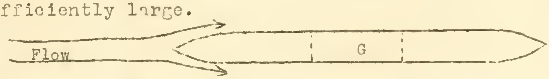


FIGURE 3. Baffle around gauge G.

The baffle could extend across the shock tube making the flow effectively two dimensional and therefore particularly amenable to calculation of pressure distribution. (Presumably for measuring air blast in the open the size of the baffle would

be limited by buffeting effects.)

It is the purpose of this memorandum to estimate the pressure perturbation corresponding to flow around various indicative shapes of baffles. It is assumed that an effectively steady flow around the gauge is attained. Actually there are reflections from the side walls of the tube subsequent to the initial scattering of the shock wave by the gauge and its baffle, but these transients are apparently rapidly attenuated, yielding nearly steady flow with or without stationary shock waves depending on the Mach number* of the flow. A typical pressure-time curve(3 at the gauge shows an unsteady pressure immediately after the shock passes, followed by a nearly constant pressure until the reflection waves from the tube ends reach the gauge.

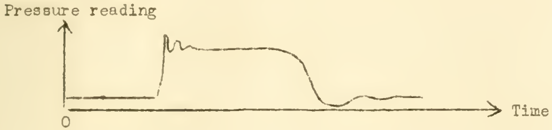


FIGURE 4. Typical pressure-time record from an exposed gauge in the shock tube.

It is further assumed that the walls of the tube are sufficiently distant from the gauge to have no appreciable effect. (See p.13 Appendix for criterion.)

The following types of calculations are possible:

(i) Acoustic scattering (Wave Equation) if the product terms in the hydrodynamical equations are negligible (which is the case if the gauge tends to be non-directional as its size is reduced). Apart from the analytical complexity, this type of calculation would not be restricted to the steady state hypothesis.

(ii) Incompressible, irrotational steady flow (potential flow) around various baffle shapes.

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* That is, the unperturbed flow speed (relative to the gauge) divided by the speed of sound (calculated for the temperature in the flowing medium behind the shock front).

(iii) Compressible, irrotational, steady, subsonic flow around the same baffle shapes using the Prandtl-Glauert rule, modified for large velocities by the results of Kaplan [4].

(iv) Compressible, irrotational, steady, supersonic flow, using the results of Donov [5].

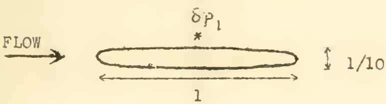
(v) Turbulent or unsteady flow.

Here we report only on (ii), (iii) and (iv). We calculate the percentage pressure perturbation

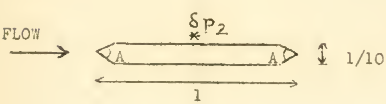
$\delta p = \dfrac{-\Delta p}{p}$ x 100 to be expected for various baffle shapes,

and estimate the thickness required to make $\delta p \sim 1 - 5\ \%$.

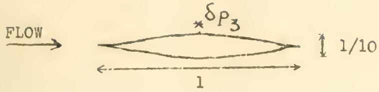
3. RESULTS The figures given in tables I and II below summarize the results of calculations based on the formulas given in the appendix. The pressures p_0, p, p_1 are as indicated in Fig. 2, M is the Mach number of the flow (behind the shock wave), and the δp's are the percentage decreases in p (at the * marked positions shown in the following cases) due to the placing of the baffles in the flow. Four different shapes of baffles are considered in Table I, the first three being infinite cylinders ("two dimensional" cases) with cross-sections as illustrated below, and the fourth being an oblate spheroid ("three dimensional" case):



Elliptical cylinder.



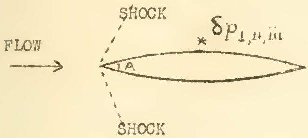
Flat double wedge cylinder (angle A between 90° and 180° for calculations in table I).



Smooth "bump" cylinder (Kaplan's type [6]).

Oblate spheroid (axes 1 and 1/10) (three dimensional example).

In Table II the δp's denote the corresponding percentage pressure decreases due to lens shaped infinite cylinders in supersonic flows, the subscripts i, ii and iii referring to A = 90°, 60° and 45° respectively (see Figure below):



Supersonic flow around a sharp nosed convex cylinder. According to Donov[5] the pressure at * DEPENDS ONLY ON the angle A.

In both tables ΔT denotes the temperature jump thru the shock front. The δp's over-estimate the percent perturbations in gauge readings since the gauges average the pressure distributions over their pick up faces, and the * marked points correspond to the largest perturbations in the region near the middle of the baffle. For example, the pressure perturbation around an elliptical cylindrical baffle placed "edge on" in a uniform incompressible flow varies according to the factor

$$1 - \frac{(a+b)^2 \sin^2\theta}{a^2 \sin^2\theta + b^2\cos^2\theta}$$

where a and b are the semi-axes (a>b) of the ellipse having the parametric equations $x = a \cos\theta$, $y = b \sin\theta$. In fact for <u>low Mach</u> numbers it would be advisable to place the gauge near a position on the baffle where this factor is zero. Unfortunately the factor is completely different near and beyond the Mach number unity.

Table I (Subsonic flow)

p/p_o	p_1/p_o	M	δp_1	δp_2	δp_3	δp_4	ΔT^o(C) *(in air)*
1.3	1.7	.18	.5%	.3%	.9%	.4%	21°
1.6	2.6	.32	1.6	1.0	2.8	1.2	40°
1.9	3.9	.44	3.1	1.9	5.3	2.5	57°
2.2	5.4	.52	5.0	3.1	8.5	3.8	74°
2.5	7.3	.62	7.1	4.4	12.1	5.4	90°
2.8	11.0	.69	9.5	5.9	18	7.2	105°
3.1	13.5	.75	13.6	7.7	25	9.5	120°
3.4	16.1	.80	19	10.8	35	13.3	135°

Table II (Supersonic flow)

p/p_o	p_1/p_o	M	δp_i	δp_{ii}	δp_{iii}	ΔT^o(C) *(in air)*
10	170	1.34	25%	5.8%	3.2%	440°
12	1300	1.42	28	6.4	3.5	530°
14	2900	1.47	27	6.3	3.4	630°
20	33000	1.58	27	6.1	3.3	900°

The Mach number M in the above tables should be referred to sound speed at the temperature $T_o + \Delta T$, where T_o is the initial temperature of the medium around the baffle.

4. Remarks.

The above values of δp should be quite accurate for a shock tube subject to size conditions given on p. 13. For actual blast waves in open air (or water)* it is possible that there may be additional sizable contributions due to the rapid variation of pressure behind the shock front. When (i) p. 3 does not apply it is apparently almost impossible to handle such contributions theoretically. Further study is planned.

* For water p is replaced by p + 3000 atmos and γ by 7, in the formulas in the Appendix. In the tables columns 3-7 apply to steady plane shocks in any medium.

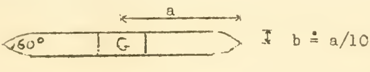
The low values of δp for the flat double wedge indicate that for subsonic flow it is important to have an appreciable part of the baffle flat around the face of the gauge.

For supersonic flow, the edge angle A should be as small as is practical.

Comparing the results for the ellipsoid and the corresponding elliptic cylinder, it is suggested that for a three dimensional baffle obtained by rotating a given profile about its minor axis, δp will be roughly 3/4 the δp for the corresponding two-dimensional flow around the profile.

In general, the dependence of δp on t is approximately linear for small ratios t of thickness to length of baffle. For a baffle of a given shape but twice as thick, p will be nearly twice as large. For values of t greater than .3, however, δp varies considerably over the gauge face and hence an integrated mean must be employed.

The following baffle shape should reduce δp to less than 5 percent, for all values of M except those near 1 .



SIDE VIEW OF BAFFLE

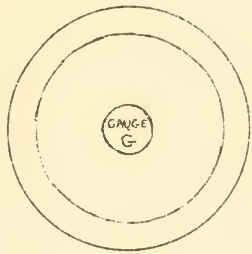


FIGURE 5. Recommended form of baffle. TOP VIEW OF BAFFLE

To reduce δp to 1 percent for values of $M \sim .7$ or .8 it would be necessary to have $b/a \sim 1:50$.

For an oblate spheroidal baffle the actual perturbation in the gauge reading can be calculated fairly accurately - a fact which is of interest when the size of the baffle is restricted by buffeting effects.

Appendix

a) Pressure relations in the shock tube.

Using the notations and results of G. T. Reynolds,[7] we have

(1) $$\frac{p_1}{p_o} = \frac{p}{p_o}\left\{1 - \frac{(p/p_o) - 1}{\sqrt{7 + 42(p/p_o)}}\right\}^{-7} .$$

This is the formula used in calculating p_1/p_o. It is valid only when the expansion and conpression chambers are in thermal equilibrium before the diaphragm is broken. In case there is an initial temperature discontinuity, $T_1 - T_o$,

(1') $$\frac{p_1}{p_o} = \frac{p}{p_o}\left\{1 - \frac{(p/p_o) - 1}{\sqrt{(7 + 42\ p/p_o)T_1/T_o}}\right\}^{-7} .$$

b) Mach number of the flow.

If M is the Mach number of the mass flow behind the shock wave when no gauge is present, then

(2) $$M^2 = \frac{(1 - p_o/p)^2}{.28 + 1.68\ p_o/p} .$$

This may be obtained directly from the following relations:[8]

(3) $$\frac{\rho}{\rho_o} = \frac{\mu^2 p_o + p}{\mu^2 p + p_o}$$

(4) $$(u - w)^2 = \frac{1}{\rho^2}\left\{-\frac{p_o - p}{\frac{1}{\rho_o} - \frac{1}{\rho}}\right\}$$

(5) $$w^2 = \frac{1}{\rho_o^2}\left\{-\frac{p_o - p}{\frac{1}{\rho_o} - \frac{1}{\rho}}\right\}$$

(6)
$$M^2 = \frac{\rho u^2}{\gamma p}$$

(7)
$$\mu^2 = \frac{\gamma - 1}{\gamma + 1} \quad ; \quad \gamma = 1.4 \quad \text{(for air)}$$

When

w is the velocity of the shock front

u is the velocity of the air behind the shock front

ρ, ρ_o are the densities of the air behind and in front of the shock front, respectively.

c. <u>The percentage pressure deviation (subsonic)</u>

Using the notation of Kaplan (9, we define a pressure coefficient,

(8)
$$C_{p,M} = \frac{\Delta p}{\frac{1}{2}\rho u^2} ,$$

where Δp is the change in the pressure p due to the presence of the gauge and baffle. It is assumed that the flow pattern around the baffle is steady, the air compressible, and that the walls of the tube have no effect.

By the Prandtl-Glauert rule (10 for thin profiles,

(9)
$$C_{p,M} = \frac{C_{p,o}}{\sqrt{1 - M^2}}$$

where $C_{p,o}$ is the pressure coefficient for incompressible flow - i.e. a shape factor. From the Bernoulli equation

(10)
$$C_{p,o} = 1 - \left(\frac{q}{u}\right)^2$$

where q is the velocity of flow across the gauge (nearly constant since the gauge is small with respect to the baffle). Combining these, we have

(11)
$$\delta_p = \frac{-70\ M^2}{\sqrt{1 - M^2}} C_{p,o} . \quad (70 \text{ is } 50\ \gamma)$$

The extensive calculations of Kaplan indicate that this formula is correct for the shapes considered for values of M up to .6. For higher values of M it gives a result which is slightly too small, depending on the values of M and t. The procedure of this memorandum has been to use (11), but multiply it by a weighting factor for M ~ .7 or .8, the weighting factor depending on the relative thickness and upon M. This factor never exceeds 1.3 in the present work.

d. <u>The pressure coefficient $C_{p,o}$ (subsonic).</u>

This must be calculated directly from the incompressible flow pattern around the baffle.

(i) <u>Elliptic cylinder</u>: axes 2a, 2at .

The two-dimensional flow pattern around an ellipse is well known[11]. At the end of the semi-minor axis, i.e. at the middle of the gauge , it gives:

$$q = u\,(1 + t)\ . \tag{12}$$

Hence, referring to (10)

$$C_{p,o} = -2t - t^2\ . \tag{13}$$

For t = .1, this gives

$$C_{p,o} = -.21\ . \tag{14}$$

(ii) <u>Flat, tapered wedge.</u>

For the two-dimensional flow around the flat wedge pictured on page 4. the standard Schwartz-Christoffel mapping method[12] gives

(15) $q = u\left(1 + \frac{2}{\pi} t + \text{ higher power in } t\right)$

at the mid point. To the first power in t, q is independent of the (given) vertex angle. Again choosing $t = .1$, we obtain

(16) $$C_{p,o} = .13.$$

(iii) Smooth 'bump'. (Kaplan's type)[13]

The parametric equations of this profile are

(17)
$$x = \frac{2+t}{4} \cos \theta - \frac{t}{4} (3 \cos \theta - \cos 3\theta)$$
$$y = \frac{t}{4} (3 \sin \theta - \sin 3\theta) \quad .$$

The two-dimensional flow around this baffle has been calculated by Kaplan[14] and gives, for $t = .1$,

(18) $$C_{p,o} = - .36 .$$

Actually, since this shape is tapered to a cusp at the ends, it is relatively thicker for a given value of t than, say, an ellipse. For $t = .06$, this shape is roughly equivalent to an ellipse with $t = .1$.

(iv) Ellipsoid: axes 2a, 2a, 2at .

To estimate the error introduced by two-dimensionalizing the flow, the pressure coefficient for an ellipsoid is calculated. The velocity potential of the incompressible flow pattern around an ellipsoid is[15]

(19) $$\phi = ux\left\{1 + \frac{a^3 t}{2-\alpha_0} \int_\lambda^\infty \frac{d\sigma}{(a^2+\sigma)^2 (a^2t^2+\sigma)^{1/2}}\right\}$$

when λ is defined by

(20) $$\frac{x^2 + y^2}{a^2 + \lambda} + \frac{z^2}{a^2 t^2 + \lambda} = 1 ,$$

and

(21) $$\alpha_o = a^3 t \int_0^{\infty} \frac{d\sigma}{(a^2 + \sigma)^2 \, (a^2 t^2 + \sigma)^{1/2}} .$$

At the point (o, o, at), this gives

(22) $$q = u(1 + \frac{\alpha_o}{2 - \alpha_o}) .$$

α_o may be integrated explicitely, and gives, for $t = .1$, $\alpha_o = .14$. Hence,

(23) $$C_{p,o} = - .16 .$$

e) The percentage pressure perturbation (supersonic).

The results obtained by Donov[16] may be used directly to give

(24) $$\delta p = f(M)\,\beta^3 ,$$

when 2β is the edge angle, A(see page 5) and

(25) $$f(M) = \frac{25\gamma(\gamma+1)M^6}{(M^2-1)^{7/2}} \left\{ - \frac{1}{3} + \frac{3-\gamma}{6} M^2 + \frac{3\gamma - 5}{24} M^4 \right\} .$$

The gauge surface is assumed to be flat and the baffle should be convex. The flow has been two dimensionalized.

f) Temperature variation.

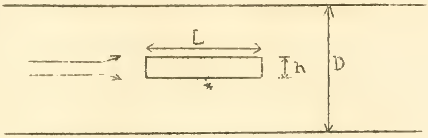
Assuming the gas law pv= RT, and using (3), we obtain

(26) $$\Delta T = T_o \frac{(p/p_o)^2 - 1}{1 + 6p/p_o} ,$$

when T_o is the temperature in the expansion chamber in front of the shock, and $T_o + \Delta T$ is the temperature behind the shock.

g) <u>Tube wall effect.</u>

To estimate the error involved in neglecting the tube walls the two dimensional flow around a rectangle in a channel was compared with that of the same rectangle in an infinitely wide channel.



The velocity of flow at the mid point* of the rectangle can be calculated by means of the Schwartz-Christoffel mapping process. Assuming that h is small in comparison with L, the result is,

$$q^2 = u^2 \left(1 + \frac{4t}{\pi} \left(1 + \frac{s^2 \pi^2}{4}\right) - \cdots \right)$$

where $s = L/D$. The percentage error in neglecting the wall effect, then is

$$\delta C_{p,o} = 100 \frac{C_{p,o}(s) - C_{p,o}(s=o)}{C_{p,o}(s = o)}$$

(27)

$$= 25\pi^2 s^2$$

This formula is valid for small s only. For $s \sim .2 - .4$ it gives too large a value for the error. A successive approximation calculation for $s = .4$ and $t = .1$ was carried out and indicated a value of $\delta C_{p,o}$ of somewhat less than 10 percent. For a baffle whose length is less than 40 percent of the shock tube diameter, then, the effect of the tube wall will not materially change the results in Tables I and II. This result refers to $M \ll 1$. For $M \geqq 1$ the walls should produce negligible perturbation if $L/(D - h) < M$.

Footnotes

(1 "A preliminary study of plane shock waves formed by bursting diaphragms in a tube," G. T. Reynolds, NDRC Report A-192(OSRD-1519) June 1943.

(2. "Characteristics of Air Blast Gauges" Arons , Tait, Fraenkel and Deane, NDRC Report AES-8a (OSRD-4875a) March 1945

(3. See (1 page 29

(4. "The flow of a compressible fluid past a curved surface," Carl Kaplan, ARR#3K02, Nov. 1943

(5. "A plane wing with sharp edges in a supersonic stream" Donov (in Russian) Izvestia Akad. Nauk SSSR (Serie Math.) 1939

(6. This is the surface considered by Kaplan, loc.cit. For an equation of the surface see Appendix

(7. See (1 page 10

(8. "Supersonic flow and shock waves," AMP Report 38.2R, page 93

(9. See (4 page 12

(10. See (4 page 12

(11. "Theoretical Hydrodynamics," Milne-Thomson, page 159

(12. See (11 Chapter X

(13. See (4 page 17

(14. See (4 page 73

(15. See (11 page 456

(16. See (5

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